A Phase-Locking Method for Beam Steering in Active Array Antennas

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Abstract—**A new principle for phase shifting, and hence beam steering, on an active array antenna is described. Each individual RF source is phase locked by a stable locking signal which is close in frequency to the nth harmonic of its free-running frequency. Pulses of appropriate amplitude and duration applied to the dc circuitry cause the fundamental output frequency of the RF source to shift in phase by increments of $2\pi/n$. The construction and testing of a four-element L-band (1-GHz) array using transistor oscillators locked at S-band (4 GHz) to give 90° phase increments is described.**

I. INTRODUCTION

ONE of the more common methods of electronic beam steering in array antennas is that of phase scanning in which the RF signal from a high-power source is split by means of power dividers and fed through phase shifters to individual radiating elements. Beam steering is achieved by means of control signals applied to the phase shifters which then cause an approximately linear phase variation of the RF signal across the array, thereby launching a wavefront in the required angular direction. Continuously variable, or analog, phase shifters may be used, but in general digital phase shifters offer advantages in lightness, speed of operation, and ease of interfacing with control computers. For a digital phase shifter, the size of the phase increments needed is determined by the radar design requirements—in particular the number of radiating elements, element spacing, and scan-angle increment. For large arrays with typically a hundred or more elements the phase increments may be coarse, and 2-bit (90°-increment) rather than 3-bit (45°-increment) or 4-bit ($22\frac{1}{2}$°-increment) phase shifters may be used. A method for the elimination of large numbers of passive phase shifters was given by Mailloux *et al.* [1] in proposing an IF scanning system for a passive array. The method used a series of harmonically related frequencies of identical phase to produce identical frequency signals separated by multiples of a phase increment.

Much recent work with array antennas has been devoted to active arrays in which each radiating element has associated with it an individual source of RF power. Such a system is particularly suitable for use with solid-state microwave devices in view of the limited power presently available from an individual device. The individual sources may either be acting as amplifiers or oscillators, and in both cases a reference or locking signal must be provided to each device to maintain the phase coherence between the outputs.

The present paper describes a novel method of beam steering for an active array using locked oscillators. Phase shifters are not needed since, by locking the oscillators with a reference signal harmonically related to the fundamental output, a dual function of locking and phase shifting may be achieved. The elimination of passive phase shifters brings obvious advantages at the expense of providing extra microwave circuitry at the harmonic frequency. It is immediately obvious that harmonic locking will become progressively more difficult as the harmonic number increases and the method probably finds optimum application for low harmonic numbers. Particular attention will therefore be paid to fourth-harmonic locking which yields a 2-bit or 90°-increment system.

II. THE METHOD OF INCREMENTAL PHASE LOCKING

Since oscillators are strongly nonlinear devices, locking of the oscillator output can take place for injected signals close in frequency to harmonics of the free-running output. A new technique of achieving digital phase shifting by the use of harmonic locking was suggested by Cullen [2]. An oscillator running at frequency f and locked by a reference signal at the frequency nf can be shifted in phase by π/n as a result of reversing the reference-signal phase. Reversal of reference-signal phase at fourth harmonic, for example, produces a phase shift of ±45° on the fundamental output of the locked oscillator. The direction of the phase shift is determined largely by the relationship between the oscillator free-running frequency and the reference frequency. If the reference phase is reversed in a short time compared with the oscillator time constant, the oscillator is momentarily unlocked and phase advance will be obtained if the oscillator runs fast with respect to the reference until relocking occurs. Thus, by using a phase-reversal switch in conjunction with a harmonically locked oscillator, it was proposed that the cost and complexity of conventional digital phase-shifting systems could be reduced.

This principle leads to the consideration of a similar scheme, eliminating the phase-reversal switch, but resulting in a phase shift now of $2\pi/n$. An oscillator of free-

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running frequency f_0 is locked to an injected nth harmonic signal at frequency nf, where $f \simeq f_0$; the frequency f_0 is then changed sufficiently to drive the oscillator out of lock; if the change in frequency is maintained for sufficient time to allow the oscillator to slip by one nth of a cycle at fundamental frequency (i.e., one cycle at the harmonic frequency) and then removed, the oscillator free-running frequency will return to f_0 and locking will take place immediately again at the frequency f. The oscillator will then be running at a phase of $\pm 2\pi/n$ relative to its previous locked phase. The sense of this phase shift is determined by the direction in which the free-running frequency is shifted; a positive frequency shift results in a phase lead, and vice versa. This method enables a constant reference to be used, with phase lead or lag obtained by controlling the free-running frequency of the oscillator.

The oscillator free-running frequency change to cause unlocking could be obtained by a change in the oscillator circuit conditions; for example, with a varactor. It is often, however, more convenient to vary the oscillator bias. The amplitude of pulse (ΔV) applied to the oscillator dc bias circuitry controls the magnitude of frequency shift and hence the rate of change of phase in the locked state; the pulsewidth (T) controls the length of time for which this rate of change is maintained and therefore the total phase change. Positive and negative pulses will yield opposite frequency shifts so the direction of phase shift is given simply by pulse polarity. A useful feature of this method is that the pulse parameters are not critical. For a phase shift of $2\pi/n$ between successive locked states to be obtained, it is necessary that the control pulse should give rise to a phase shift of more than π/n and less than $3\pi/n$; between these limits the oscillator will pull in and lock with the required $2\pi/n$ phase change. This indicates that a departure of up to ± 50 percent on the product $T \cdot \Delta V$ is acceptable.

The relationship between frequency shift (Δf) and pulse duration is simply given by $T\Delta f = 1/n$ for a phase shift of $\pm 2\pi/n$. Insertion of some specimen quantities shows that an oscillator locked at fourth-harmonic frequency can, for a frequency shift of 10 MHz, be incremented in phase by 90° in a time of 25 ns. The phase shifting is therefore rapid by comparison with most existing methods.

In order to assess this phase-shifting method further, it is important to consider the basic properties of harmonic locking and the associated power requirements.

III. GENERAL PRINCIPLES OF HARMONIC LOCKING

Microwave oscillators can be represented by the equivalent circuit of a nonlinear admittance in parallel with a resonant circuit. A useful simplification for the purposes of analytical treatment is to consider the nonlinear element as a pure nonlinear negative conductance. Many fundamental locking analyses have been carried out with this circuit configuration [3], [4] as shown in Fig. 1.

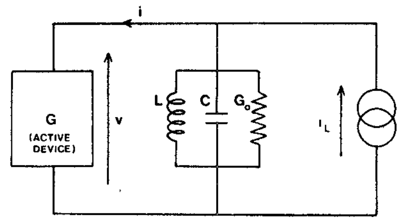

Fig. 1. Circuit for simplified locking analyses.

The describing equation for the circuit is

$$Gv + G_0 v + C\frac{dv}{dt} + \frac{1}{L}\int v\,dt = i_L$$

or

$$C\ddot{v} + \left(G + v\frac{dG}{dv} + G_0\right)\dot{v} + \frac{1}{L}v = \frac{di_L}{dt}. \qquad (1)$$

The conductance G can be expressed as a polynomial

$$G = -a_1 + a_2 v + a_3 v^2 + a_4 v^3 \qquad (2)$$

to third order in voltage. It is sufficient to truncate the polynomial at this point where harmonics of order four or less are being considered. With some normalization, this leads to the equation

$$\ddot{v} + (b_1 + b_2 v + b_3 v^2 + b_4 v^3)\dot{v} + \omega_0^2 v = \frac{1}{C}\frac{di_L}{dt} \qquad (3)$$

where

$$b_1 = (G_0 - a_1)/C$$

$$b_2 = 2a_2/C, \qquad b_3 = 3a_3/C, \qquad b_4 = 4a_4/C$$

$$\omega_0^2 = 1/LC.$$

In the case of fundamental locking, the locking source provides a current i_L:

$$i_L = I_L \cos \omega t \qquad (4)$$

and the circuit voltage would be given for locked conditions as

$$v = V \cos(\omega t - \phi). \qquad (5)$$

Differentiating, expanding the terms in powers of v as harmonics of the fundamental frequency ω, and collecting terms in $\sin \omega t$ and $\cos \omega t$ yields two equations:

$$(\omega_0^2 - \omega^2)V = \frac{-\omega I_L}{C}\sin\phi \qquad (6)$$

$$b_1 V + b_3\frac{V^3}{4} = \frac{I_L}{C}\cos\phi. \qquad (7)$$

Under conditions where the detuning is small ($\omega_0 - \omega = \Delta\omega \ll \omega_0$), the locking range is given by taking the limiting values of $\sin\phi = \pm 1$. Thus

$$\Delta\omega \simeq \pm \frac{\omega_0 I_L}{2CV} = \pm \frac{\omega_0}{2Q} \cdot \frac{I_L}{G_0 V} = \pm \frac{\omega_0}{2Q} \left(\frac{P_1}{P_0}\right)^{1/2} \tag{8}$$

where Q is the loaded Q factor of the oscillator. This equation is often known as Adler's equation [5] and is often put in terms of injected (P_1) and output (P_0) powers since these quantities are easier to measure at microwave frequencies.

In the case of harmonic locking for an nth harmonic signal, the simplest situation to analyze is that in which the current generator at frequency ω is replaced by one at frequency $n\omega$. Then

$$i_L = I_L \cos n\omega t. \tag{9}$$

The circuit voltage is now given by

$$v = V \sin(\omega t - \phi_1) + V_n \cos n\omega t. \tag{10}$$

A case of particular interest is that of fourth-harmonic locking ($n = 4$), which yields 90°-phase increments. As in the previous analysis, two equations result from collecting terms in $\cos \omega t$ and $\sin \omega t$ from the previous relationships substituted into the describing equation. To first order in V_4, these equations are

$$(\omega_0^2 - \omega^2) - \frac{b_4}{8} V^2 V_4 \omega \sin 4\phi = 0 \tag{11}$$

$$b_1 + b_3 \frac{V^2}{4} + b_4 \frac{V^2}{8} V_4 \cos 4\phi = 0. \tag{12}$$

The first of these equations, under conditions of small detuning, as before, yields an expression for the locking range

$$\Delta\omega \simeq \pm \frac{\omega_0}{2Q} \left(\frac{a_4 V^3}{2G_0}\right) \frac{V_4}{V} = \pm \frac{\omega_0}{2Q} \left(\frac{a_4 V^3}{2G_0}\right) \left(\frac{P_4}{P_0}\right)^{1/2}. \tag{13}$$

By comparison with the expression obtained in the case of fundamental locking, it is possible to obtain an estimate of the relative locking powers at fundamental (P_1) and harmonic (P_4) frequency which would be required for the same locking range. Thus

$$\frac{P_4}{P_1} = \frac{4G_0^2}{a_4^2 V^6}. \tag{14}$$

It may be noted that the relative locking power is inversely proportional to the square of the fourth-order nonlinear coefficient for the negative conductance—a not unexpected result. It follows, therefore, that the locking power required at harmonic frequencies depends strongly on the nonlinear characteristics of the device.

A theoretical analysis of a 1-GHz transistor oscillator, the detailed explanation of which is not presented in the present paper, provided an analytic form for the equivalent nonlinear negative conductance from which the polynomial coefficients a_{1-4} were determined. Substitution of numerical values in (14) yielded a value of P_4/P_1 close to +10 dB. The actual measurements on a 1-GHz transistor oscillator are shown in Fig. 2, and indicate that the simple theory represents reasonably well the practical situation.

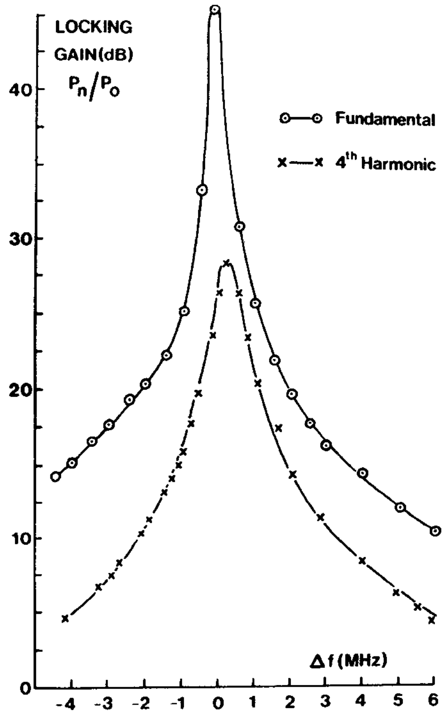

Fig. 2. Comparison of locking gains at fundamental and fourth-harmonic frequency for a transistor oscillator.

An increase of locking power by 10 dB over that required for fundamental locking was not felt to be unduly serious since locking gains of 10 dB were still easily obtainable for stable oscillator operation over a 0.5-percent locking bandwidth.

IV. THE EXPERIMENTAL ACTIVE ARRAY

A. The RF Sources

The frequency chosen for the active array was 1 GHz. The most convenient active microwave devices for low-power oscillators at this frequency are transistors, and there is the advantage that considerable work has been carried out on the design and packaging of such transistor oscillators at L band in stripline and coaxial-line circuits. The main design problem in the present work was to devise means for coupling the external harmonic signal into the oscillator. A configuration similar to the coaxial microwave triode oscillator was found to be appropriate. A commercial transistor (BFY 90) suitable for low-power (~10-mW) oscillator circuits in the region of 800–1200 MHz was chosen. The oscillator design is shown in Fig. 3, and consists of two coaxial lines of 50-Ω characteristic impedance with movable short-circuiting plungers. The transistor is situated in the center plate with its base grounded, and the inner conductors of the two coaxial cavities are connected to the emitter and collector through feed-through capacitors; the dc bias circuit is connected directly to the emitter and collector. The generated power is coupled out by an adjustable potential probe in the collector cavity, while a similar probe in the emitter cavity serves for the input of the harmonic signal. These two

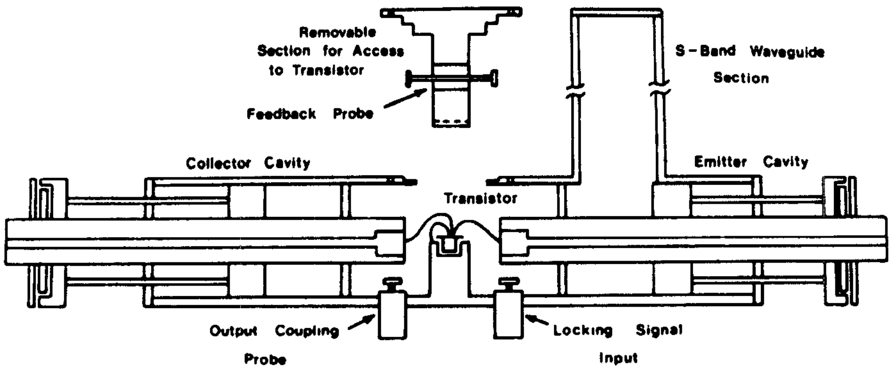

Fig. 3. Experimental 1-GHz transistor oscillator.

probes are situated very close to the transistor so as to be at the points of maximum potential in the cavities. An S-band waveguide section, having a cutoff frequency above the fundamental oscillator frequency is coupled to the emitter cavity to facilitate tuning to the externally injected harmonic signal independently of tuning to the fundamental operating frequency. A potential probe between the emitter and collector cavities provides the positive feedback necessary for oscillation.

B. Locking and Phase-Increment Tests

The transistor oscillator is an ideal RF source for the intended phase-shift application. The steady-state working point of the transistor can be controlled with small changes in base-emitter voltage. In order to calculate the characteristics of the control pulse required for the phase incrementation, it is necessary to determine the oscillator free-running frequency variation with changes in base-emitter voltage. This is shown in Fig. 4. Also shown in the figure are curves obtained when locking signals of different magnitude are applied. As expected, well outside the locking range, the frequency deviation obtained with a given base voltage change approaches the free-running value. For voltage changes so that the free-running frequency is not driven beyond the locking-range limits, the oscillator frequency does not change, as expected. The symmetry of these results shows that there is approximately the same magnitude of frequency shift for similar voltage changes of opposite polarity. This confirms the possibility of phase shifting in positive or negative directions depending on control-pulse polarity.

The required control-pulse characteristics may easily be calculated from Fig. 4. For a 90°-phase increment, as described earlier, a relationship $T\Delta f = \frac{1}{4}$ applies. For a given time T in which the phase shift has to be accomplished, the control pulsewidth is fixed and equal to T; the required frequency shift Δf is thus equal to $1/4T$, and the corresponding voltage shift, or control pulse height, can be read from Fig. 4. An example serves to illustrate: for a phase increment of 90° to be accomplished in 250 ns, Δf is 1 MHz; with 15-dB locking gain, this frequency shift is accomplished with a base voltage change of just over 100 mV; the required control pulse is therefore of height 100 mV and width 250 ns.

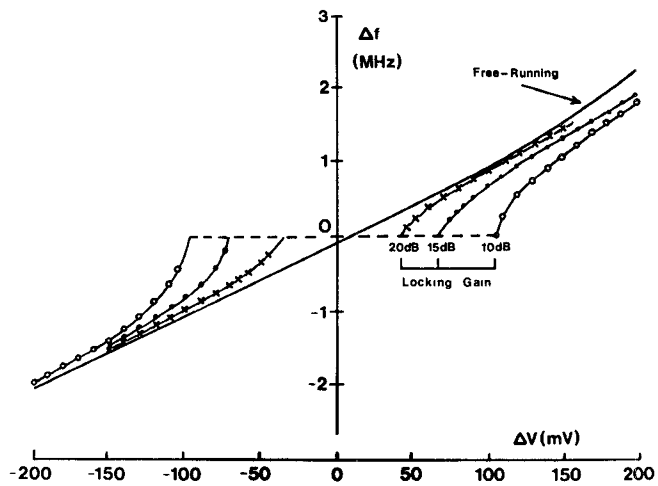

Fig. 4. Dependence of oscillator frequency output on base voltage change in free-running and harmonically locked modes.

The limiting factors on the control-pulse parameters are as follows: for very slow phase shifting where T is large and the frequency shift is small, it is very difficult to control the base voltage change to sufficient accuracy since the oscillator is being driven marginally beyond the locking range. For very rapid phase shifting where T is small, the base voltage change must be large, and the limiting factor is the voltage change that can be applied without damage to the transistor. The frequency dependence on base voltage only shows a somewhat linear variation for small changes in the base voltage. Large changes can cause the transistor to be driven into a nonoscillating mode.

The phase-shifting operation was demonstrated successfully for a wide range of control-pulse characteristics by varying T and ΔV, but keeping the product $T\Delta V$ approximately constant. It was found as expected that neither the pulse shape nor the values of T and ΔV were critical. For dc isolation, the control pulses were supplied to the transistor from a pulse transformer, the secondary of which was in series with the base connection. The shortest phase-shifting time used was 40 ns, corresponding to a shift of about 6 MHz in the oscillator frequency during phase shifting. The curve of Fig. 4 and the values previously quoted are all for a fixed oscillator Q (measured value $\sim$25). It follows therefore that the faster phase shifting could be obtained with a lower Q oscillator, but at the expense of lower locking gain for equivalent frequency stability in locked operation.

C. *Antenna-Array Design and Tests*

Slot radiating elements were chosen for ease of alignment and support. The feed points from rigid coaxial line were at 1.5 cm from the end of each slot and gave measured VSWR readings less than 1.12 in all cases. Four transistor oscillators, each with an individual connection to a pulse generator for supplying control pulses, were connected directly to the slots. The mutual coupling between the slots at fundamental frequency was reduced in two ways: by introducing vanes of approximately half a wavelength between the slots, and by inserting an isolator in each feed between the oscillator and slot. The total effect of these two measures was to reduce the mutual coupling between adjacent slots to -35 dB. With a harmonic locking gain of 15 dB, the effects of mutual coupling at fundamental frequency were then small and satisfactory phase shifting of the signal fed to the individual slots was obtained. The complete system is shown in Fig. 5.

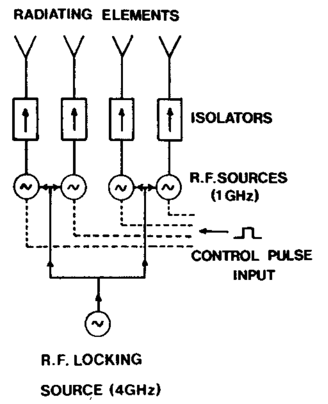

Fig. 5. Block diagram of complete system.

For measurement of the antenna radiation pattern the array was placed on a rotating turntable in an anechoic chamber. The receiving antenna was a stationary dipole at a distance just in the far field. The pattern obtained is shown in Fig. 6 for two phase combinations on the elements; one is the broadside position with all elements in phase, and the other shows a shifted position with the end elements radiating with phase shifts of $+90°$ and $-90°$, respectively.

D. *Array Radiation-Pattern Calculation*

The array pattern was calculated using array theory and allowing for mutual coupling [6], [7]. Since oscillator output levels were not identical, measured amplitudes were used in the pattern calculation. If allowance is made for the phase errors in initial setting up, taking a value of $\pm 20°$ for the purpose of calculation, satisfactory agreement is obtained between theory and experiment for the radiation patterns in the broadside and shifted positions. The theoretical curves are shown as broken lines in Fig. 6. All the possible combinations of $\pm 90°$ and $\pm 180°$ phase shifts were tested, and in each case the radiation patterns demonstrated beam steering and satisfactory agreement with theoretical radiation patterns. The phase shifts of 180° were obtained by applying two consecutive control pulses of the same polarity, thereby enabling the array to be operated with one characteristic control pulse.

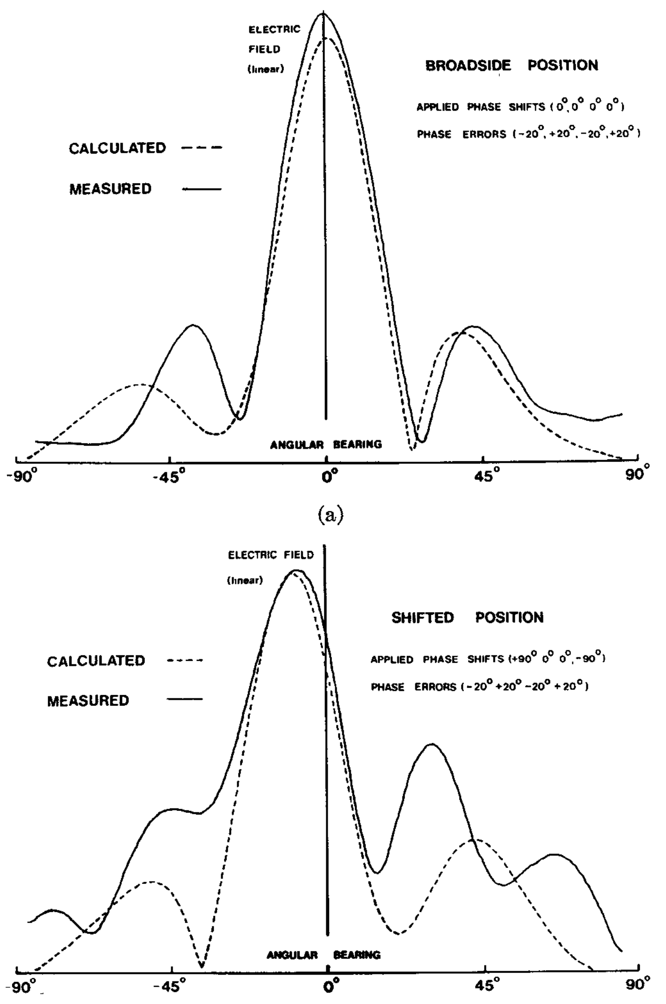

Fig. 6. (a) Antenna radiation pattern, broadside position. (b) Antenna radiation pattern, shifted position.

V. DISCUSSION

A novel method of beam steering for active array antennas has been tested and described. The array tested comprised too small a number of elements for practical purposes, as shown by the high sidelobe levels, but it served to demonstrate the principle of beam forming and steering by the use of harmonic locking.

The method has some advantages over existing phase-shift techniques. Its prime advantages are that the need for passive phase shifters is eliminated, that an exact phase increment is always produced, and that the control circuits are fundamentally simple; there is a large tolerance to control pulse characteristics and phase-shifting times can be made very short. The disadvantage of the method is the need to supply locking power at a harmonic of the fundamental output frequency instead of at the fundamental frequency itself, together with associated microwave circuitry at the harmonic frequency. The amount of harmonic locking power needed is a strong function of the nonlinear active device characteristics, and it was found in the case of a transistor oscillator that the locking power required at fourth harmonic was about 10 dB greater than that required at fundamental frequency. For higher harmonics, the locking power will be still larger in general, and it is therefore felt that the method has application for low-order harmonics ($n \lesssim 4$) and for radar systems with fundamental frequencies at or below X band. This limits the phase-shift increment to 90° as the smallest possible. This is almost certainly satisfactory for large arrays, and work is in hand on an alternative method of locking whereby the effective phase shift per element would be half this value.

In any practical system it is necessary to incorporate a means of setting the initial phase of all elements. In the present experiments this occurred through the small amount of mutual coupling at fundamental frequency between the individual sources via the radiating elements. The level of coupling was such that it did not prevent phase shifting under harmonic-locking conditions, but locked all oscillators in phase in the absence of harmonic signal input. An alternative method would be to replace the isolators in Fig. 5 with circulators. The third port of the circulator would provide for injection of a synchronizing signal during initial beam formation. It is likely that some simple waveguide or coaxial structure could be devised to serve the purposes of coupling in a weak synchronizing signal for initialization and for coupling out a weak signal to monitor the phase-change increments.

Some consideration must also be given to the question of phase errors. Although the phase-shift increment is always exactly $2\pi/n$, phase errors in the initial state do arise unless all oscillators receive identical locking power and have identical free-running frequencies. These phase errors cannot exceed $\pm 22\frac{1}{2}°$ in the case of fourth-harmonic locking since this corresponds to $\pm 90°$ at the fundamental frequency and is the limit of the locking range. The phase errors can be reduced away from the value of $22\frac{1}{2}°$ by accurate tuning of the oscillators and an adequate level of locking power; this ensures that all the free-running frequencies lie well within the locking range of an individual oscillator and that the phase errors will be considerably less than the maximum value previously given.

The phase-shifting principle, though described in detail for RF sources using transistors, is completely general and could be used, for example, with IMPATT-diode oscillators. The work carried out so far relates principally to a CW radar system, but there is scope for consideration of applications to pulse radar and to complete transmit and receive systems.

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